Formulas for Cooling With Exhaust Fans

BY MIRZA M. SHAH, PH.D., P.E., FELLOW/LIFE MEMBER ASHRAE

Exhaust fans drawing outdoor air are widely used for cooling industrial and commercial facilities. Air drawn into the room picks up the room heat before leaving through the exhaust fans. Examples are electrical rooms, boiler rooms, manufacturing shops, etc. Consider a room with sensible heat gain H with an exhaust fan with volumetric flow rate Q. Outside air at temperature T_{OA} enters the room, picks up heat to rise to a temperature T_{ROOM} and is exhausted by the fan. Neglecting the moisture in air, which has very little effect as its quantity is very small, heat balance gives:

$$T_{ROOM} = T_{OA} + \frac{H}{C_p D_{ROOM} Q} \tag{1}$$

Where D_{ROOM} is the density of air leaving the room and C_p is the specific heat of air. During the analysis and evaluation of existing systems, it is often required to find the room temperature that will be maintained by an exhaust system with a fixed flow rate. To calculate the room temperature T_{ROOM} with Eq. (1), the density of air D_{ROOM} is required. This density depends on room temperature, which is unknown. Thus tedious trial and error calculations are needed. Calculations are often simplified by assuming the room air density to be the same as the outside air density or that at standard conditions. The approximate formula most commonly used according to *ASHRAE Handbook*[1] is:

$$H = 1.1\, Q\, (T_{ROOM} - T_{OA}) \tag{2}$$

This can result in considerable error if the temperature rise in room is large. To enable easy calculation of room temperature without trial and error, the author has developed the formulas presented here. Formulas are also given for easier accurate calculation of required fan capacity.

As most calculations are done at sea level, formulas for sea level are given below. Their derivation is given in Appendix 1 of this article. Formulas for use at other elevations are given in Appendix 2.

Formulas for Room Temperature

In customary I-P units, the formula is:

$$T_{ROOM} = \frac{572 T_{OA} Q}{572 Q - H} \tag{3}$$

In SI units, the formula is:

$$T_{ROOM} = \frac{0.353 T_{OA} Q}{0.353 Q - H} \tag{4}$$

These formulas are derived from Eq. (1) by using the

Mirza M. Shah, Ph.D., P.E., is director of Engineering Research Associates. He is active in research, design, and analysis in mechanical engineering, including heat transfer, fluid flow, HVAC, refrigeration, plumbing, fire protection, energy conservation, and renewable energy systems.

Nomenclature

C_p = Specific heat of air at constant pressure, Btu/lb R (J/kg K)

D = Density of air, lb/ft^3 (kg/m^3)

H = Sensible heat load of space being cooled, Btu/h (kW)

P = Absolute atmospheric pressure at a location, psia (Pa)

P_{SL} = Absolute atmospheric pressure at sea level, psia (Pa)

p^* = $=p/p_{SL}$

Q = Airflow at exhaust fan inlet, cfm (L/s)

T = Temperature, R (K)

Subscripts

OA = Outside air

ROOM = Room air

ST = Standard air

perfect gas equation and re-arrangement (Appendix 1). They give exactly the same result as Eq. (1).

Example One

An electrical room has a sensible heat gain of 500,000 Btu/h (146 536 W). It is provided with an exhaust fan of 10,000 cfm (4719 L/s) capacity on the west wall. Air intake louvers are installed on the east wall. Outside air is at 90°F (32°C) dry bulb and 30% relative humidity. Calculate the room temperature.

$$T_{OA} = 90 + 460 = 550 \text{ R}$$

Using Present Formula

Using Eq. (3),

$$T_{ROOM} = 572 \times 550 \times 10{,}000/(572 \times 10{,}000 - 500{,}000) = 603 \text{ R} = 143°\text{F}$$

By Trial and Error

If Eq. (2) is used as is the usual practice, T_{OA} is calculated to be 135°F (57°C).

The next step for trial and error is to calculate the air density at 135°F (57°C). Plotting sensible heat gain from 90°F (32°C) DB and 30% RH on psychrometric chart as a horizontal line to 135°F (57°C), air density is found to be 0.066 lb/ft^3 (1 kg/m^3). Note that air density can also be calculated with software such as ASHRAE Duct Fitting Database (Air Properties). Using Eq. (1) with this density, room temperature is calculated to be 142.6°F (61.4°C). This is close to the correct value given by Eq. (3) but the person doing calculations does not know it until further iterations converge. Therefore, one more calculation using the air density at 142.6°F (61.4°C) will be done to confirm that the correct value of room temperature has been found. In many cases, more than two iterations may be needed.

It is thus clear that the new formula will significantly reduce the effort needed for accurate calculations.

Example Two

A diesel-generator room has a heat gain of 200 kW at maximum load and 100 kW at part load. It is provided with an exhaust fan of 10,000 L/s (21 189 cfm) at the roof and air intake fans at the walls. Outdoor air temperature is 30°C (86°C). Calculate the room temperature at maximum and part loads.

$$T_{OA} = 30 + 273 = 303 \text{ K}$$

Applying Eq. (4),
At full load,

$$T_{ROOM} = 0.353 \times 10{,}000 \times 303/(0.353 \times 10{,}000 - 200) = 321.2 \text{ K} = 48.2°\text{C}$$

At part load,

$$T_{ROOM} = 0.353 \times 10{,}000 \times 303/(0.353 \times 10{,}000 - 100) = 312 \text{ K} = 39°\text{C}$$

Formulas for Exhaust Fan Capacity

For calculating fan capacity, Eq. (1) may be re-arranged to:

$$Q = \frac{H}{C_p Q D_{ROOM} (T_{ROOM} - T_{Air})} \quad (5)$$

To use this formula, the density of room air D_{ROOM} has to determined. While it can be done quickly with a psychrometric chart, to get an accurate value this way is difficult as it requires interpolation between constant specific volume lines. For accurate value of density, one must use psychrometric tables and formulas or software such as that in the ASHRAE Duct Fitting Database; this is tedious. The following formulas allow calculation of required fan capacity without the need to calculate room air density; their derivation is given in Appendix 1.

In I-P units:

$$Q = \frac{H}{572(1 - T_{OA} / T_{ROOM})} \quad (6)$$

In SI units:

$$Q = \frac{H}{0.353(1 - T_{OA} / T_{ROOM})} \quad (7)$$

Example Three

A mechanical room has sensible heat gain of 120,000 Btu/h (35 169 W). The room temperature should not exceed 104°F (40°C). Design outdoor temperature is 86°F (30°C). Calculate required capacity of exhaust fan.

$$T_{ROOM} = 104 + 460 = 564 \text{ R}$$

$$T_{OA} = 86 + 460 = 546 \text{ R}$$

Using Eq. (6),

$$Q = 120{,}000/[572 \times (1 - 546/564)] = 6{,}573 \text{ cfm}$$

Conclusions

- A formula has been given with which room temperature can be determined easily and accurately for rooms cooled by exhaust fans without tedious trial and error calculations, which would otherwise be needed.
- Another formula has been given with which the required capacity of exhaust fan for space cooling can be easily and accurately determined without the tedious calculations which are needed for accurate determination of air density.

References

1. ASHRAE Fundamentals Handbook 2017, page 18.15.

Appendix One

Derivation of Formulas for Sea Level

Using the perfect gas law, density of air at any temperature can be related to the standard air density as below.

$$D_{ROOM} = D_{ST} \times T_{ST} / T_{ROOM} \quad (1A1)$$

Substituting in Eq. (1),

$$T_{ROOM} = T_{OA} + \frac{H}{C_p Q D_{ST} T_{ST} / T_{ROOM}} \quad (2A1)$$

Re-arranging,

$$T_{ROOM} = \frac{T_{OA} C_p D_{ST} T_{ST} Q}{T_{OA} C_p D_{ST} T_{ST} Q - H} \quad (3A1)$$

In IP units, C_p = 0.24 Btu/lb °F. D_{ST} = 0.075 lb/ft^3 at T_{ST} = 69°F = (69 + 460) = 529 R.[1] Airflow is usually expressed in cfm and H in Btu/h. Hence Q is to be multiplied by 60. Substituting these values in Eq. (3A1), Eq. (3) is obtained.

In SI units, C_p = 1 kJ/kg K, D_{ST} = 1.2 kg/m^3 at T_{ST} = 21°C = (273 + 21) = 294 K. Airflow is usually expressed in L/s hence Q has to be divided by 1,000. Substituting these values in Eq. (3A1), Eq. (4) is obtained.

Re-arrangement of Eqs. (3) and (4) yields Eqs. (6) and (7).

Appendix Two

Formulas for Other Elevations

Using the perfect gas equation, the density of air at elevations other than sea level may be calculated by:

$$D = D_{SL} p / p_{SL} \quad (1A2)$$

Where the subscript "SL" means at sea level.

Inserting this relation in Eq. (3A1) with $p^* = p/p_{SL}$:

$$T_{ROOM} = \frac{T_{OA} C_p D_{ST} p^* T_{ST} Q}{C_p D_{ST} p^* T_{ST} Q - H} \quad (2A2)$$

By substituting the values as in Appendix 1, the following formulas are obtained.

In customary I-P units,

$$T_{ROOM} = \frac{572 T_{OA} Q p^*}{572 Q p^* - H} \quad (3A2)$$

In SI units,

$$T_{ROOM} = \frac{0.353 T_{OA} Q p^*}{0.353 Q p^* - H} \quad (4A2)$$

In I-P units,

$$Q = \frac{H}{572 p^* (1 - T_{OA} / T_{ROOM})} \quad (5A2)$$

In SI units,

$$Q = \frac{H}{0.353 p^* (1 - T_{OA} / T_{ROOM})} \quad (6A2)$$